



MATHEMATICS

## STOCHASTIC MODELS ON TIME TO RECRUITMENT IN A TWO GRADE MANPOWER SYSTEM USING DIFFERENT POLICIES OF RECRUITMENT

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### Abstract

In this paper a two grade organization in which depletion of manpower occurs due to its policy decisions is considered. Two mathematical models are constructed employing two different univariate recruitment policies, based on shock model approach. The mean and variance of the time to recruitment are obtained for both the models under different conditions. The analytical results are numerically illustrated and relevant conclusions are presented.

**Keywords:** Two grade system, Shock models, Univariate policies of recruitment, Mean and variance of the time to recruitment

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### Introduction

Exit of personnel is quite common in any marketing organization when it takes policy decisions such as revision of targets, emoluments etc. Frequent recruitment is costlier and since the number of exits is probabilistic, a suitable recruitment policy has to be designed on time to recruitment, otherwise the organization will reach the breakdown point. In [1], [2], [3], [5], [6], [7], [9], [10], [11] for a two grade system, employing a univariate cum policy of recruitment in which recruitment is done as and when the cumulative loss of manpower crosses a threshold for the organization, performance measures namely mean and variance of time to recruitment are obtained assuming different conditions on loss of manpower, nature of thresholds and inter decisions times. In [4] most of the above cited results are also derived using a univariate max policy of recruitment in which recruitment is done as and when the maximum loss of manpower crosses the thresholds for the organization. The objective of the present paper is to obtain the above cited performance measures under a more general setting. To this end, two mathematical models are constructed, one employing the univariate cum policy of recruitment and other using univariate max policy of recruitment. Influence of nodal parameters on performance measures is also analyzed for these models through a numerical example.

### Model description and analysis for model -I

Consider an organization with two grades I and II taking decisions at random epochs in  $[0, \infty)$ . At every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. Let  $V_i(t)$  be the probability that there are exactly  $i$  decisions in  $[0, t)$ . It is assumed

that loss of manhours  $X_{1i}$  and  $X_{2i}$  in grades I and II respectively for decision  $i$ , form a sequence of independent and identically distributed exponential random variables with parameters  $\lambda_1$  and  $\lambda_2$  ( $\lambda_1, \lambda_2 > 0$ ). For  $i = 1, 2, 3, \dots$  let  $X_i = \max(X_{1i}, X_{2i})$  be the loss of manhours in the organizations due to  $i^{\text{th}}$  decision and

$g(\cdot)$  be its density function. Let  $S_i = \sum_{j=1}^i X_j$  be the

cumulative loss of manhours in the first  $i$  decisions and  $g_i(\cdot)$  be its density function. It is assumed that the inter decision times are independent and identically distributed exponential random variables. Let  $F(\cdot)$  ( $f(\cdot)$ ) be its distribution (density) function with parameter  $\theta$  ( $\theta > 0$ ). Let  $f_i(\cdot)$  ( $F_i(\cdot)$ ) be the  $i$ -fold convolution of  $f(\cdot)$  ( $F(\cdot)$ ).

Let  $\bar{z}(\cdot)$  be the Laplace transform of  $z(\cdot)$ .

The loss of manpower process and the process of inter decisions times are assumed to be statistically independent. Let  $Y_1, Y_2$  be the threshold for the loss of manhours in grades I and II respectively and  $Y$  be a suitably defined threshold for the organization. Let  $H(\cdot)$  be the distribution function of  $Y$ . For all  $i = 1, 2, 3, \dots$  it is assumed that  $X_i$  and  $Y$  are independent. It is assumed that  $Y_1, Y_2, X_{1i}$  and  $X_{2i}$  for each  $i$ , are independent. **In this model recruitment is done whenever cumulative loss of manhours crosses the threshold level  $Y$ .** Let  $W$  be the time to recruitment in the organization and  $L(\cdot)$  ( $l(\cdot)$ ) be its distribution (density) function. Let  $E(W)$  and  $V(W)$  be the mean and variance of time to recruitment.

### Main result

The survival function of  $W$  is given by

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$P(W>t) = \sum_{i=0}^{\infty}$  {Probability that there are exactly  $i$  decisions in  $[0,t)$  and cumulative loss of manhours does not crosses the threshold level  $Y$  in these  $i$  decisions}

By the law of total probability

$$P(W > t) = \sum_{i=0}^{\infty} V_i(t)P(S_i < Y) \tag{1}$$

$$= \sum_{i=0}^{\infty} V_i(t) \int_0^{\infty} [1 - H(x)]g_i(x)dx$$

From Renewal theory[8] it is known that

$V_i(t) = F_i(t) - F_{i+1}(t)$  with

$$F_0(t) = 1 \tag{2}$$

Since  $X_{1i}$  and  $X_{2i}$  follow exponential distribution with parameter  $\lambda_1$  and  $\lambda_2$  we find that

$$g(x) = \lambda_1 e^{-\lambda_1 x} + \lambda_2 e^{-\lambda_2 x} - (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)x} \tag{3}$$

**Case(i)  $Y = \max(Y_1, Y_2)$**

**Subcase(i)**

**Suppose  $Y_1$  and  $Y_2$  follow exponential distribution with parameters  $\mu_1$  and  $\mu_2$  respectively.**

In this case it can be shown that

$$\int_0^{\infty} [1 - H(x)]g_i(x)dx = [D_1]^i + [D_2]^i - [D_3]^i \tag{4}$$

where

$$D_1 = \bar{g}(\mu_1) = \frac{\lambda_1}{\mu_1 + \lambda_1} + \frac{\lambda_2}{\mu_1 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_1 + \lambda_1 + \lambda_2}$$

$$D_2 = \bar{g}(\mu_2) = \frac{\lambda_1}{\mu_2 + \lambda_1} + \frac{\lambda_2}{\mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_2 + \lambda_1 + \lambda_2} \tag{5}$$

$$D_3 = \bar{g}(\mu_1 + \mu_2) = \frac{\lambda_1}{\mu_1 + \mu_2 + \lambda_1} + \frac{\lambda_2}{\mu_1 + \mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_1 + \mu_2 + \lambda_1 + \lambda_2}$$

From(1),(2) , (4) and on simplification we get

$$L(t) = 1 - P(W > t) = A_1 + A_2 - A_3 \tag{6}$$

$$\bar{l}(s) = \bar{a}_1 + \bar{a}_2 - \bar{a}_3$$

where for  $j=1,2,3,$

$$A_j = A_j(t) = [1 - D_j] \sum_{i=1}^{\infty} F_i(t)[D]^i$$

$$\bar{a}_j = \bar{a}_j(s) = [1 - D_j] \sum_{i=1}^{\infty} f(s)[D]^i \tag{7}$$

Since  $\bar{f}(s) = \frac{\theta}{\theta + s}$  ,  $E(W) = -[\frac{d}{ds} \bar{l}(s)]_{s=0}$  and

$$E(W^2) = [\frac{d^2}{ds^2} \bar{l}(s)]_{s=0} \tag{8}$$

From (6),(7) and (8) and on simplification we get

$$E(W) = \frac{1}{\theta} \{B_1 + B_2 + B_3\} \tag{9}$$

and

$$E(W^2) = \frac{1}{\theta^2} \{B_1^2 + B_2^2 + B_3^2\} \tag{10}$$

where  $B_j = \frac{1}{[1 - D_j]}$  , $j=1,2,3.$  and  $D_1, D_2$  and  $D_3$  are

given by (5)

When  $Y_1, Y_2$  are exponential random variables (9) gives the mean time to recruitment.

From (9) and (10) the variance of the time to recruitment can be computed for this case.

**Subcase(ii)**

**Suppose distributions of  $Y_1$  and  $Y_2$  have SCBZ property**

In this case

$$P(Y_m \leq x) = 1 - p_m e^{-(\mu_{m1} + \alpha_m)x} - q_m e^{-(\mu_{m2}x)}$$
 , $m=1,2.$

Proceeding as above we get

$$\bar{l}(s) = q_1 \bar{a}_1 + q_2 \bar{a}_2 + p_1 \bar{a}_3 + p_2 \bar{a}_4 - p_1 q_2 \bar{a}_5 - p_2 q_1 \bar{a}_6 - q_1 q_2 \bar{a}_7 - p_1 p_2 \bar{a}_8 \tag{11}$$

where  $\bar{a}_n = \bar{a}_n(s) = [1 - E_n] \sum_{i=1}^{\infty} [\bar{f}(s)]^i [E_n]^{i-1}$  ,

$$n=1,2,3,4,5,6,7,8 \tag{12}$$

and

$$E_1 = \bar{g}(\mu_{12}) = \frac{\lambda_1}{\mu_{12} + \lambda_1} + \frac{\lambda_2}{\mu_{12} + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{12} + \lambda_1 + \lambda_2}$$

$$E_2 = \bar{g}(\mu_{22}) = \frac{\lambda_1}{\mu_{22} + \lambda_1} + \frac{\lambda_2}{\mu_{22} + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{22} + \lambda_1 + \lambda_2}$$

$$E_3 = \bar{g}(\mu_{11} + \alpha_1) = \frac{\lambda_1}{\mu_{11} + \alpha_1 + \lambda_1} + \frac{\lambda_2}{\mu_{11} + \alpha_1 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{11} + \alpha_1 + \lambda_1 + \lambda_2}$$

$$E_4 = \bar{g}(\mu_{21} + \alpha_2) = \frac{\lambda_1}{\mu_{21} + \alpha_2 + \lambda_1} + \frac{\lambda_2}{\mu_{21} + \alpha_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{21} + \alpha_2 + \lambda_1 + \lambda_2}$$

$$E_5 = \bar{g}(\mu_{11} + \mu_{22} + \alpha_1) = \frac{\lambda_1}{\mu_{11} + \mu_{22} + \alpha_1 + \lambda_1} + \frac{\lambda_2}{\mu_{11} + \mu_{22} + \alpha_1 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{11} + \mu_{22} + \alpha_1 + \lambda_1 + \lambda_2}$$

$$E_6 = \bar{g}(\mu_{21} + \mu_{12} + \alpha_2) = \frac{\lambda_1}{\mu_{21} + \mu_{12} + \alpha_2 + \lambda_1} + \frac{\lambda_2}{\mu_{21} + \mu_{12} + \alpha_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{21} + \mu_{12} + \alpha_2 + \lambda_1 + \lambda_2}$$

$$E_7 = g(\mu_2 + \mu_2) = \frac{\lambda_1}{\mu_2 + \mu_2 + \lambda_1} + \frac{\lambda_2}{\mu_2 + \mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_2 + \mu_2 + \lambda_1 + \lambda_2}$$

$$E_8 = g(\mu_{11} + \mu_{21} + \alpha_1 + \alpha_2) = \frac{\lambda_1}{\mu_{11} + \mu_{21} + \alpha_1 + \alpha_2 + \lambda_1} + \frac{\lambda_2}{\mu_{11} + \mu_{21} + \alpha_1 + \alpha_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_{11} + \mu_{21} + \alpha_1 + \alpha_2 + \lambda_1 + \lambda_2} \quad (13)$$

From(11),(12) and (13)

$$E(W) = \frac{1}{\theta} \{q_1 C_1 + q_2 C_2 + p_1 C_3 + p_2 C_4 - p_1 q_2 C_5 - p_2 q_1 C_6 - q_1 q_2 C_7 - p_1 p_2 C_8\} \quad (14)$$

$$E(W^2) = \frac{2}{\theta^2} \{q_1 C_1^2 + q_2 C_2^2 + p_1 C_3^2 + p_2 C_4^2 - p_1 q_2 C_5^2 - p_2 q_1 C_6^2 - q_1 q_2 C_7^2 - p_1 p_2 C_8^2\}$$

where  $C_n = \frac{1}{[1 - E_n]}$  and  $E_n$ ,  $n=1,2,3,4,5,6,7,8$  are

given by (13).

When the distributions of  $Y_1$  and  $Y_2$  have SCBZ property (14) gives the mean time to recruitment. From(14) and (15) the variance of the time to recruitment can be computed for this case.

**Subcase(iii)**

**Suppose  $Y_1$  and  $Y_2$  follow extended exponential distribution with scale parameters  $\mu_1$  and  $\mu_2$  respectively and shape parameter 2.**

$$P(Y_1 \leq x) = (1 - e^{-\mu_1 x})^2$$

$$\text{and } P(Y_2 \leq x) = (1 - e^{-\mu_2 x})^2$$

In this case it can be shown that

$$E(W) = \frac{1}{\theta} \{2(B_1 + B_2 + B_4 + B_5) - 4B_3 - B_6 - B_7 - B_8\} \quad (16)$$

$$E(W^2) = \frac{2}{\theta^2} \{2(B_1^2 + B_2^2 + B_4^2 + B_5^2) - 4B_3^2 - B_6^2 - B_7^2 - B_8^2\} \quad (17)$$

$$\text{where } B_n = \frac{1}{[1 - D_n]}, n=1,2,3,4,5,6,7,8 \quad (18)$$

and  $D_1, D_2$  and  $D_3$  are given by (5) and

$$D_4 = g(2\mu_1 + \mu_2) = \frac{\lambda_1}{2\mu_1 + \mu_2 + \lambda_1} + \frac{\lambda_2}{2\mu_1 + \mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{2\mu_1 + \mu_2 + \lambda_1 + \lambda_2}$$

$$D_5 = g(\mu_1 + 2\mu_2) = \frac{\lambda_1}{\mu_1 + 2\mu_2 + \lambda_1} + \frac{\lambda_2}{\mu_1 + 2\mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{\mu_1 + 2\mu_2 + \lambda_1 + \lambda_2}$$

$$D_6 = g(2\mu_1) = \frac{\lambda_1}{2\mu_1 + \lambda_1} + \frac{\lambda_2}{2\mu_1 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{2\mu_1 + \lambda_1 + \lambda_2}$$

$$D_7 = g(2\mu_2) = \frac{\lambda_1}{2\mu_2 + \lambda_1} + \frac{\lambda_2}{2\mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{2\mu_2 + \lambda_1 + \lambda_2}$$

$$D_8 = g(2\mu_1 + 2\mu_2) = \frac{\lambda_1}{2\mu_1 + 2\mu_2 + \lambda_1} + \frac{\lambda_2}{2\mu_1 + 2\mu_2 + \lambda_2} - \frac{\lambda_1 + \lambda_2}{2\mu_1 + 2\mu_2 + \lambda_1 + \lambda_2} \quad (19)$$

When the thresholds are extended exponential random variables (16) gives mean time to recruitment. From (16) and (17) the variance of the time to recruitment can be computed for this case.

**Case(ii)  $Y = \min(Y_1, Y_2)$**

**Subcase(i)**

**Suppose  $Y_1$  and  $Y_2$  follow exponential distribution with parameters  $\mu_1$  and  $\mu_2$  respectively.**

Then

$$P(W > t) = \sum_{i=0}^{\infty} V_i(t) [D_3]^i$$

Computing  $L(t), l(t)$  and  $\bar{l}(s)$  as in case(i) it can be shown that

$$E(W) = \frac{1}{\theta[1 - D_3]} \quad (20)$$

$$\text{and } V(W) = [E(W)]^2$$

where  $D_3$  is given by (5).

When the thresholds are exponential random variables (20) give mean and variance of the time to recruitment for this case.

**Subcase(ii)**

**Suppose the distributions of  $Y_1$  and  $Y_2$  have SCBZ property.**

In this case it can be shown that

$$E(W) = \frac{1}{\theta} \{p_1 q_2 C_5 + p_2 q_1 C_6 + q_1 q_2 C_7 + p_1 p_2 C_8\} \quad (21)$$

$$E(W^2) = \frac{2}{\theta^2} \{p_1 q_2 C_5^2 + p_2 q_1 C_6^2 + q_1 q_2 C_7^2 + p_1 p_2 C_8^2\} \quad (22)$$

where  $C_r = \frac{1}{[1 - E_r]}$  and  $E_r$ ,  $r = 5,6,7, 8$  are given by (13)

When distributions of  $Y_1$  and  $Y_2$  have SCBZ property (21)

gives the mean time to recruitment.

From(21) and (22) the variance of the time to recruitment can be computed for this case.

**Subcase(iii)**

**Suppose  $Y_1$  and  $Y_2$  follow extended exponential distribution with scale parameters  $\mu_1$  and  $\mu_2$  respectively and shape parameter 2.**

In this case it is found that

$$E(W) = \frac{1}{\theta} \{4B_3 - 2(B_4 + B_5) + B_8\} \tag{23}$$

$$E(W^2) = \frac{2}{\theta^2} \{4B_3^2 - 2(B_4^2 + B_5^2) + B_8^2\} \tag{24}$$

where  $B_3, B_4, B_5$  and  $B_6$  are given by (18).

When the thresholds are extended exponential random variables (23) gives mean time to recruitment.

From (23) and (24) the variance of the time to recruitment can be computed for this case.

**Model description for model II**

In this model two types of univariate max recruitment policies are employed. In cases (i) and (ii) recruitment is done when maximum loss of manhours in the organization crosses a constant threshold say a for the organization. But in case (iii) recruitment is made whenever either maximum loss of manhours in grade I crosses the constant threshold  $c_1$  or maximum loss of manhours in grade II crosses the constant threshold  $c_2$  whichever is earlier. In this model, the mean and variance of time to recruitment are obtained for geometric as well as exponential loss of manpower for all the three cases. All other assumption and notations are as in model I except for the structural variation in the loss of manpower for the organization.

**Case(i)**  $X_i = \max(X_{1i}, X_{2i})$

Proceeding as in model I and on simplification we get  $L(t) = 1 - P(W > t) = K(t)$ ;

$$\text{and } \bar{l}(s) = \bar{k}(s) \tag{25}$$

$$\text{where } K(t) = [1 - g(a)] \sum_{i=1}^{\infty} F_i(t) [g(a)]^{i-1} \tag{26}$$

From (25) and (26) we get

$$E(W) = \frac{1}{\theta[1 - g(a)]} \text{ and } V(W) = [E(W)]^2 \tag{27}$$

**Subcase(i)**

**Suppose  $X_{1i}, X_{2i}$  follow geometric distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.**

In this case

$$g(a) = 1 - (\lambda_1^*)^{a+1} - (\lambda_2^*)^{a+1} + (\lambda_1^* \lambda_2^*)^{a+1} \text{ where}$$

$$\lambda_m^* = 1 - \lambda_m, m=1,2. \tag{28}$$

Using (28) in (27) we get

$$E(W) = \frac{1}{\theta[(\lambda_1^*)^{a+1} + (\lambda_2^*)^{a+1} - (\lambda_1^* \lambda_2^*)^{a+1}]} \text{ and}$$

$$V(W) = [E(W)]^2 \tag{29}$$

When the loss of manpowers in each grade is geometric (29) give mean and variance of the time to recruitment for this case.

**Subcase(ii)**

**Suppose  $X_{1i}, X_{2i}$  follow exponential distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.**

In this case

$$g(a) = 1 - e^{-\lambda_1 a} - e^{-\lambda_2 a} + e^{-(\lambda_1 + \lambda_2)a}$$

$$E(W) = \frac{1}{\theta[e^{-\lambda_1 a} + e^{-\lambda_2 a} - e^{-(\lambda_1 + \lambda_2)a}]} \text{ and } V(W) = [E(W)]^2 \tag{30}$$

When the loss of manhours in each grade is exponential (30) give mean and variance of the time to recruitment for this case.

**Case(ii)**  $X_i = X_{1i} + X_{2i}$

**Subcase(i)**

**Suppose  $X_{1i}, X_{2i}$  follow geometric distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.**

In this case

$$g(a) = 1 + \frac{1}{(\lambda_2 - \lambda_1)} [\lambda_1^* (\lambda_2^*)^{a+2} - (\lambda_1^*)^{a+2} \lambda_2^*]$$

where  $\lambda_m^* = 1 - \lambda_m, m=1,$

$$E(W) = \frac{\lambda_2 - \lambda_1}{\theta[(\lambda_1^*)^{a+2} \lambda_2 - \lambda_1 (\lambda_2^*)^{a+2}]} \text{ and } V(W) = [E(W)]^2 \tag{31}$$

When the loss of manpowers in each grades is geometric, (31) give mean and variance of the time to recruitment for this case. When the loss of manhours in each grade is geometric.

**Subcase(ii)**

**Suppose  $X_{1i}, X_{2i}$  follow exponential distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.**

In this case

$$g(a) = 1 - e^{-\lambda_2 a} + \frac{\lambda_2 e^{-\lambda_1 a}}{\lambda_1 - \lambda_2} - \frac{\lambda_2 e^{-\lambda_2 a}}{\lambda_1 - \lambda_2}$$

$$E(W) = \frac{\lambda_1 - \lambda_2}{\theta[\lambda_1 e^{-\lambda_2 a} - \lambda_2 e^{-\lambda_1 a}]} \text{ and } V(W) = [E(W)]^2 \tag{32}$$

When the loss of manhours in each grades is exponential, (32) give the mean and variance of the time to recruitment for this case.

**Case(iii)**

$$S_{1i} = \max_{1 \leq j \leq i} (X_{1j}) \text{ and } S_{2i} = \max_{1 \leq j \leq i} (X_{2j})$$

For the new univariate recruitment policy mentioned in the description of the present model for case (iii) the survival function of W is given by

$$P(W>t) = \sum_{i=0}^{\infty} \{ \text{Probability that there are exactly } i \text{ decisions in } [0,t) \text{ and maximum loss of manhours in grade I does not cross } c_1 \text{ and maximum loss of manhours in grade II does not cross } c_2 \}$$

$$= \sum_{i=0}^{\infty} V_i(t) P(S_{1i} < c_1) P(S_{2i} < c_2)$$

$$\text{ie, } P(W>t) = \sum_{i=0}^{\infty} V_i(t) [g(c_1)g(c_2)]^i$$

Proceeding as in model I we get

$$\bar{l}(s) = \bar{a}(s) \text{ and}$$

$$E(W) = \frac{1}{\theta [1 - g(c_1)g(c_2)]}$$

$$\text{and } V(W) = [E(W)]^2 \tag{33}$$

$$\text{where } \bar{a}(s) = [1 - g(c_1)g(c_2)] \sum_{i=1}^{\infty} [f(s)]^i [g(c_1)g(c_2)]^{i-1}$$

**Subcase(i)**

Suppose  $X_{1i}, X_{2i}$  follow geometric distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.

In this case

$$g(c_1)g(c_2)$$

$$= 1 - (\lambda_1)^{c_1+1} - (\lambda_2)^{c_2+1} + (\lambda_1)^{c_1+1} (\lambda_2)^{c_2+1}$$

$$\text{where } \lambda_m^* = 1 - \lambda_m, m=1,2. \tag{34}$$

Use (34) in (33) we get

$$E(W) = \frac{1}{\theta [(\lambda_1^*)^{c_1+1} + (\lambda_2^*)^{c_2+1} - (\lambda_1^*)^{c_1+1} (\lambda_2^*)^{c_2+1}]}$$

$$\text{and } V(W) = [E(W)]^2 \tag{35}$$

When the loss of manpowers is geometric (35) give mean and variance of the time to recruitment for this case.

**Subcase(ii)**

Suppose  $X_{1i}, X_{2i}$  follow exponential distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively.

In this case

$$g(c_1)g(c_2) = 1 - e^{-\lambda_1 c_1} - e^{-\lambda_2 c_2} + e^{-(\lambda_1 c_1 + \lambda_2 c_2)}$$

$$E(W) = \frac{1}{\theta [e^{-\lambda_1 c_1} + e^{-\lambda_2 c_2} - e^{-(\lambda_1 c_1 + \lambda_2 c_2)}]} \text{ and}$$

$$V(W) = [E(W)]^2 \tag{36}$$

When the loss of manhours is exponential (36) give mean and variance of the time to recruitment for this case.

**Numerical Illustration**

The analytical expression for expectation and variance of the time to recruitment are analyzed by varying parameters. The influence of model parameters  $\lambda_1, \lambda_2$  and  $\theta$  on performance measures namely mean and variance of the time to recruitment for model I is shown in table-1 for case(i) and table-2 for case(ii) by varying one parameters and keeping the other parameters fixed. In table -3 and table-4 the corresponding results for model- II are shown.

**Table-1: Effect of  $\lambda_1$ ,  $\lambda_2$  and  $\theta$  on performance measures**

( $\mu_1=0.3; \mu_2=0.5; \mu_{11}=0.3; \mu_{12}=0.4; \alpha_1=0.5; \mu_{21}=0.1; \mu_{22}=0.2; \alpha_2=0.3$ )

			MODEL I					
$\lambda_1$	$\lambda_2$	$\theta$	Case(i)					
			subcase(i)		subcase(ii)		subcase(iii)	
			E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	12.2043	146.5430	14.4839	198.0793	13.3999	169.7946
0.3	0.2	0.10	14.7417	206.5923	19.1520	320.4928	17.1468	253.2937
0.5	0.2	0.10	16.0149	239.7837	21.1916	382.7577	18.9074	297.2499
0.4	0.2	0.15	10.3192	100.2295	13.5479	158.4891	12.1198	123.7846
0.4	0.4	0.15	12.5839	142.3191	17.6626	248.2323	15.4125	179.8685
0.4	0.6	0.15	14.0025	171.9768	20.0704	310.3726	17.4093	218.5137
0.6	0.5	0.10	22.7573	445.6044	33.2125	827.7532	28.6328	567.6320
0.6	0.5	0.15	15.1715	198.0464	22.1417	367.8903	19.0885	252.2809
0.6	0.5	0.20	11.3786	111.4011	16.6062	206.9383	14.3164	141.9080

**Table-2: Effect of  $\lambda_1$ ,  $\lambda_2$  and  $\theta$  on performance measures**

( $\mu_1=0.3; \mu_2=0.5; \mu_{11}=0.3; \mu_{12}=0.4; \alpha_1=0.5; \mu_{21}=0.1; \mu_{22}=0.2; \alpha_2=0.3$ )

			MODEL I					
$\lambda_1$	$\lambda_2$	$\theta$	Case(ii)					
			subcase(i)		subcase(ii)		subcase(iii)	
			E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	10.3992	108.1425	10.8592	117.7014	10.8986	118.1258
0.3	0.2	0.10	10.9663	120.2588	11.9750	142.3412	12.1060	143.1590
0.5	0.2	0.10	11.3372	128.5323	12.6276	157.7203	12.8375	158.9304
0.4	0.2	0.15	7.4468	55.4550	8.2280	67.0643	8.3433	67.5120
0.4	0.4	0.15	8.0000	64.0000	9.2660	84.1873	9.4769	84.1253
0.4	0.6	0.15	8.4000	70.5600	9.9692	96.8157	10.2570	96.2279
0.6	0.5	0.10	13.0589	170.5352	15.7841	241.3899	16.2967	238.3004
0.6	0.5	0.15	8.7059	75.9934	10.5227	107.2844	10.8645	105.9113
0.6	0.5	0.20	6.5295	42.6338	7.8921	60.3475	801483	59.5751

**Table -3: Effect of  $\lambda_1$ ,  $\lambda_2$  and  $\theta$  on performance measures when  $a=0.3$**

			MODEL II					
$\lambda_1$	$\lambda_2$	$\theta$	Case(i)				Case(ii)	
			subcase(i)		subcase(ii)		subcase(i)	
			E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	10.3330	106.7718	10.0172	100.3451	10.2983	106.0544
0.3	0.2	0.10	11.0305	121.6730	10.0504	101.0100	10.9275	119.4098
0.5	0.2	0.10	11.7583	138.2580	10.0818	101.6423	11.5979	134.5119
0.3	0.2	0.15	7.3537	54.0769	6.7003	44.8934	7.2850	53.0710
0.3	0.4	0.15	8.1305	66.1053	6.7322	45.3224	7.9883	63.8127
0.3	0.6	0.15	8.9882	80.7885	6.7625	45.7320	8.7834	77.1481
0.6	0.5	0.10	17.0479	290.6309	10.2348	104.7520	16.3759	268.1711
0.6	0.5	0.15	11.3653	129.1693	6.8232	46.5565	10.9173	119.1871
0.6	0.5	0.20	8.5239	72.6577	5.1174	26.1880	8.1880	67.0428

**Table-4: Effect of  $\lambda_1$ ,  $\lambda_2$  and  $\theta$  on performance measures**  
(a=0.3;c<sub>1</sub>=0.3; c<sub>2</sub>= 0.1)

$\lambda_1$	$\lambda_2$	$\theta$	MODEL II					
			Case(ii)		Case(iii)			
			subcase(ii)		subcase(i)		subcase(ii)	
			E(W)	V(W)	E(W)	V(W)	E(W)	V(W)
0.1	0.2	0.10	10.0087	100.1749	10.2866	105.8139	10.0059	100.1171
0.3	0.2	0.10	10.0258	100.5157	10.8785	118.3421	10.0171	100.3417
0.5	0.2	0.10	10.0422	100.8449	11.4845	131.8930	10.0277	100.5539
0.3	0.2	0.15	6.6838	44.6737	7.2523	52.5965	6.6780	44.5963
0.3	0.4	0.15	6.7004	44.8955	7.9318	62.9132	6.6892	44.7460
0.3	0.6	0.15	6.7164	45.1103	8.7216	76.0664	6.7003	44.8934
0.6	0.5	0.10	10.1225	102.4653	15.9078	253.0581	10.0810	101.6264
0.6	0.5	0.15	6.7483	45.5401	10.6052	112.4703	6.7207	45.1673
0.6	0.5	0.20	5.0613	25.6163	7.9539	63.2645	5.0405	25.4066

**We observe the following:**

(1) Decrease in the average loss of manhours delays the time to recruitment on the average in reality when all other nodal parameters are fixed. This aspect is reflected in tables 1,2,3,4.

(2) Increase in the average inter-decision times delays the time to recruitment on the average in reality when all other nodal parameters are fixed. This aspect is reflected in tables 1,2,3,4.

(3) From tables 1 and 2 ,as for as the model I is concerned case(i) gives a better options for the organization than case(ii) as the average time to recruitment for case(i) is greater than that of case(ii).

(4) From tables 3 and 4 ,as for as the model II is concerned case(i) gives a better options for the organization than cases(ii) and(iii) as the average time to recruitment for case(i) is greater than that of cases(ii) and (iii).

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