Journal of Applied and Advanced Research 2025, 10: 1-10
Doi: 10.21839/jaar.2025.v10.9559

Research Article

Statistical optimization for plant disease classification using quantum adiabatic machine learning

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(Received: January 28, 2025; Revised: April 16, 2025; Accepted: April 16, 2025; Published: April 24, 2025)
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ABSTRACT

Quantum Adiabatic Machine Learning (QAML) leverages principles of adiabatic quantum computation to optimize machine learning tasks such as plant disease classification. By encoding the problem into quantum Hamiltonians and using adiabatic evolution, QAML explores a vast solution space efficiently. This method uses quantum concepts such as entanglement to navigate complex energy landscapes that classical approaches find challenging. The framework applies both theoretical and practical insights to address high-dimensional data issues, essential for identifying plant diseases accurately and efficiently. We show that QAML optimizes classification tasks by encoding plant disease detection into an Ising-model-inspired Hamiltonian. The adiabatic process retains optimal configurations via multi-qubit entanglement while mitigating decoherence effects. Furthermore, we mathematically demonstrate QAML's resilience to noise in open quantum systems and its potential quantum advantage over classical methods. This approach promises advancements in agricultural diagnostics and quantum-enhanced learning.

Key words: QAML, Plant disease recognition, Quantum Hamiltonians, Adiabatic quantum computation, Ising model applications, Quantum noise resilience, Multi-qubit entanglement

INTRODUCTION

Classical probability theory

The probability of an event A is defined as the ratio of the number of favorable outcomes ($n(A)$) to the total number of possible outcomes ($n(S)$):

$$P(A)=\frac{n(A)}{n(S)}$$

The probability of an event A not occurring (A^{o}) is defined as

$$P(A^{\circ})=1-P(A)$$

Thus,

$$P(A)+P(A^{\circ})=1$$

For two events, A and B, the probability that either A or B occurs is

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

If A and B are mutually exclusive,

$$P(A\cup B)=P(A)+P(B)$$

For two independent events, A and B, the probability that both occur is

$$P(A\cap B)=P(A)\cdot P(B)$$

If A and B are dependent events,

$$P(A\cap B)=P(A)+P(B\,|\,A)$$

where $P(B\,|\,A)$ is the conditional probability of B, given that A has occurred. The conditional probability of event B, given that event A has occurred, is

$$P(B\,|\,A)=\frac{P(A\cap B)}{P(A)}$$

For events,

$$A_1,A_2,...,A_n$$

where one must occur,

$$P(A_i\,|\,B)=\frac{P(B\,|\,A_i)P(A_i)}{\sum_{j=1}^{n}P(B\,|\,A_j)P(A_j)}$$

This is known as Bayes theorem (Grinstead & Snell, 1997; Bertsekas & Tsitsiklis, 2008).

Quantum states

A quantum state is represented as a vector $|\Psi\rangle$ (ket notation) in a Hilbert space:

$$|\Psi\rangle=-\sum_{i}c_i|i\rangle$$

Here, $|i\rangle$ are basis vectors (eigenstates of an observable), and c_i are complex coefficients called probability amplitudes. The normalization condition ensures:

$$\langle\Psi|\Psi\rangle = 1$$

or

$$\sum_i |c_i|^2 = 1$$

Quantum mechanics allows linear superpositions of states. If

$$|\Psi_1\rangle \; and \; |\Psi_2\rangle$$

are solutions to the Schrödinger equation, any linear combination is also a valid state:

$$|\Psi\rangle = a|\phi_1\rangle + |b\phi_2\rangle$$

Where $a, b \in C$

Physical observables (e.g., position, momentum, energy) correspond to Hermitian operators (O) acting on the state vector:

$$O|\Psi\rangle = o|\Psi\rangle$$

Here, o is the eigenvalue (measured value) and $|\Psi\rangle$ is the eigenstate. The probability of measuring an eigenvalue o_i associated with eigenstate $|i\rangle$ is given by the Born rule:

$$P(o_i) = |i|\Psi\rangle|^2$$

After measurement, the system collapses to the eigenstate corresponding to the measured eigen value.

The time evolution of a quantum state is governed by the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

Here, H is the Hamiltonian operator (total energy), $\hbar$ is the reduced Planck constant. The solution is:

$$|\Psi(t)\rangle = e^{\frac{i}{\hbar}Ht}|\Psi(0)\rangle$$

For mixed states or statistical ensembles, the density matrix formalism is used:

$$\rho = -\sum_i p_i |\Psi_i\rangle\langle\Psi_i|$$

Here, p_i are probabilities

$$\left(p_i \geq 0, \sum_i p_i = 1\right)$$

for pure states $\rho = |\Psi\rangle\langle\Psi|$

For two quantum systems A and B, their combined state lies in the tensor product space:

$$|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$

This allows entanglement, where the state cannot be written as a product of individual states.

A qubit's state can be written as:

$$|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$$

Here, $|0,|1\rangle$ are the basic states, and the probabilities are:

$$P(0) = |c_0|^2, P(1) = |c_1|^2, |c_0|^2 + |c_1|^2 = 1$$

The Bloch sphere represents qubit states geometrically.

For an observable with eigenstates, $O|i\rangle = o_i|i\rangle$ any state can be expanded as:

$$|\Psi\rangle = \sum_i c_i |i\rangle$$

Where $c_i = \langle i|\Psi\rangle$

For continuous variables like position (x):

$$|\Psi\rangle = \int_{-\infty}^{+\infty} dx\Psi(x)|x\rangle$$

Here, $\Psi(x)$ is the wave function.

Operators like position (x) and momentum

$$\left(p_x = -i\hbar\frac{d}{dx}\right)$$

obey the commutation relation:

$$\left[x, p_x\right] = i\hbar$$

This leads to Heisenberg's uncertainty principle:

$$\sigma_x, \sigma_{p_x} \geq \frac{\hbar}{2}$$

These equations form the mathematical foundation of quantum states and their dynamics in quantum mechanics (Avron & Kenneth, 2020; Cuffaro & Fuchs, 2024; Medvidović & Moreno, 2024; Yao *et al.,* 2025).

Multiple qubits

A single qubit state is represented as:

$$|\Psi = \alpha|0\rangle + \beta|1\rangle$$

where

$$|\alpha|^2 + |\beta|^2 = 1$$

Here:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

are the computational basis states.

$$\alpha, \beta \in C$$

are complex probability amplitudes. For two qubits, the combined state is represented by the tensor product:

$$|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle = (\alpha_1 |0\rangle + \beta_1 |1\rangle) \otimes (\alpha_2 |0\rangle + \beta_2 |1\rangle)$$

Expanding this:

$$|\Psi\rangle = \alpha_1\alpha_2 |00\rangle + \alpha_1\beta_2 |01\rangle + \beta_1\alpha_2 |10\rangle + \beta_1\beta_2 |11\rangle$$

In vector form:

$$|\Psi\rangle = \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix}$$

The normalization condition is:

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

where α_{ij} are the coefficients of the basis states $|ij\rangle$

For three qubits, the state is a tensor product of three single-qubit states:

$$|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle \otimes |\Psi_3\rangle$$

Expanding this:

$$|\Psi\rangle = \sum_{i,j,k \in \{0,1\}} c_{i,j,k} |ijk\rangle$$

where $c_{i,j,k}$ are complex coefficients satisfying:

$$\sum_{i,j,k} |c_{i,j,k}|^2 = 1$$

The vector representation has $2^3 = 8$ components. For n qubits, the state is represented in a 2^n-dimensional Hilbert space:

$$|\Psi\rangle = \sum_{i=0}^{2^{n-1}} c_i |i\rangle$$

where $|i\rangle$ represents the computational basis states, and c_i are complex coefficients satisfying

$$\sum_{i=0}^{2^{n-1}} |c_i|^2 = 1$$

Entanglement occurs when the state of multiple qubits cannot be written as a simple tensor product of individual states. For example, the Bell state (an entangled two-qubit state) is:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

This cannot be factored into separate single-qubit states.

Quantum gates act on multi-qubit systems through matrix operations on their state vectors. The CNOT gate (two qubits) flips the second qubit (target) if the first qubit (control) is $|1\rangle$

Its matrix representation is:

$$U_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Applying this to a two-qubit state: If

$$|\Psi_{in}\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle$$

then

$$|\Psi_{in}\rangle = {}_{U_{CNOT}|\Psi_{in}\rangle}$$

The Hadamard gate creates superpositions. For n=2, the operation $H^{\otimes n}$ applies Hadamard to each qubit.

Measurement collapses the quantum state into one of its basis states. For example, the probability of measuring a specific outcome $|i, j, ..., k\rangle$ is given by:

$$P(|i, j, \ldots, k\rangle) = |c_{i,j,\ldots k}|^2$$

After measurement, the system collapses to the measured state (Chaddha, 2006; Griffiths, 2017).

Mixed states

Mixed states (Greiner, 2011; Everitt *et al.,* 2023) are a fundamental concept in quantum mechanics that extend beyond pure states. They represent statistical ensembles of quantum states and are essential for describing systems with incomplete information or entanglement. A mixed state is represented by a density matrix, ρ, defined as

$$\rho = \sum_i p_i |\Psi_i\rangle \langle\Psi_i|$$

where $|\Psi_i\rangle$ are pure states, and p_i are probabilities satisfying $0 \le p_i \le 1$ and

$$\sum_i p_i = 1$$

Densities matrices have the following properties:

(1) Hermiticity,

$$\rho = \rho^\dagger$$

(2) Positive semi-definiteness,

$$\langle\phi|\rho|\phi\rangle \ge 0 \text{ for all } |\phi\rangle$$

(3) Trace condition,

$$Tr(\rho)=1$$

(4) Purity,

$$Tr(\rho)^2 \le 1$$

with equality for pure states. Pure state: $\rho = |\Psi\rangle\langle\Psi|$

A mixed state cannot be represented by a single ket vector.

For an observable, *A*, the expectation value is

$$\langle A\rangle = Tr(\rho A) = \sum_i p_i \langle \Psi_i | A | \Psi_i \rangle$$

The von Neumann entropy quantifies the mixedness of a state:

$$S(p) = -Tr(\rho \log \rho) = -\sum_i \lambda_i \log \lambda_i$$

where λ_i are eigenvalues of ρ. The time evolution of a mixed state is governed by the von Neumann equation:

$$i\hbar \frac{\partial \rho}{\partial t} = [H, \rho]$$

where *H* is the Hamiltonian, and [*H*, ρ] is the commutator. For a bipartite system *AB*, the reduced density matrix of subsystem A is

$$\rho A = Tr_B(\rho AB)$$

The probability of measuring outcome *m* is

$$P(m) = Tr(\rho P_m)$$

Evolution in closed quantum systems

Further to time-dependent Schrödinger equation and the solution to the Schrödinger equation, for a finite-dimensional Hilbert space, the exponential of the Hamiltonian is expanded using the Taylor series (Guedes *et al.*, 2016):

$$e^{-\frac{i}{\hbar}Ht} = I - \frac{i}{\hbar}Ht - \frac{1}{2!}\left(\frac{i}{\hbar}Ht\right)^2 + \cdots$$

If *H* is time-independent, the solution to the Schrödinger equation simplifies to:

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}Ht} |\Psi(0)\rangle$$

For eigen states of $H, H|E_n\rangle = E_n |E_n\rangle$ the time evolution is:

$$|\Psi(t)\rangle = \sum_n c_n e^{-\frac{i}{\hbar}E_n t} |E_n\rangle$$

where $c_n = \langle E_n | \Psi(0)\rangle$

If *H* is a time-dependent Hamiltonian, and *H* satisfies $H = H(t)$ the time-evolution operator would satisfy:

$$i\hbar \frac{\partial U(t)}{\partial t} = H(t)U(t)$$

The formal solution is given by the time-ordered exponential:

$$U(t) = \tau\, exp\left(-\frac{i}{\hbar}\int_0^t H(t')dt'\right)$$

Where τ denotes time ordering.

In the Heisenberg picture, operators evolve in time while states remain fixed. The time evolution of an operator $O_H(t)$ is given by the Heisenberg equation:

$$i\hbar \frac{\partial O_H(t)}{\partial t} = [O_H(t), H]$$

The Ehrenfest theorem connects quantum mechanics to classical mechanics by describing how expectation values evolve:

$$\frac{\partial}{\partial t}\langle O\rangle = \frac{i}{\hbar}\langle [H, O]\rangle + \left\langle \frac{\partial O}{\partial t}\right\rangle$$

This shows that quantum expectation values follow classical equations of motion for certain operators.

For a closed system with a time-independent Hamiltonian

$$\frac{\partial H}{\partial t} = 0$$

the expectation value of energy remains constant over time:

$E = \langle H\rangle_t = \langle H\rangle_{t=0} = \langle \Psi(O)|H|\Psi(O)\rangle = const.$

Open quantum systems

The state of an open quantum system (Bahns *et al.*, 2019; Breuer & Petruccione, 2002) is described by a density matrix ρ, which is a Hermitian, positive semi-definite operator with trace 1:

$$Tr(\rho) = 1$$

For an open system interacting with its environment, the dynamics of the density matrix are generally non-unitary.

The total system (system + environment) evolves unitarily under the full Hamiltonian H_{total}:

$$H_{total} = H_S \otimes I_E + I_S \otimes H_E + H_{SE}$$

where H_S is the Hamiltonian of the system, H_E is the Hamiltonian of the environment, and H_{SE} is the interaction Hamiltonian between system and environment. The state of the total system is described by a joint density matrix $\rho_{total}(t)$ evolving as

$$\frac{\partial \rho_{total}(t)}{\partial t} = -\frac{i}{\hbar}[H_{total}, \rho_{total}(t)]$$

To describe the system alone, the environmental degrees of freedom is traced out:

$$\rho_S(t) = Tr_E(\rho_{total}(t))$$

For Markovian (memoryless) environments, the evolution of the reduced density matrix $\rho_S(t)$

is governed by the Lindblad master equation:

$$\frac{\partial \rho_S(t)}{\partial t} = -\frac{i}{\hbar}\left[H_S, \rho_S(t)\right] + \sum_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2}\left\{L_k^\dagger L_k, \rho_S(t)\right\}\right)$$

where L_k are Lindblad operators describing dissipative processes, and {..., ...} denotes the anti-commutator. The Lindblad equation ensures that: $\rho_S(t)$ remains Hermitian and positive semi-definite, and $Tr(\rho_S(t)) = 1$

Non-Markovian dynamics account for memory effects and are described by integro-differential equations. A common form is the Nakajima-Zwanzig equation:

$$\frac{\partial \rho_S(t)}{\partial t} = -i\left[H_S, \rho_S(t)\right] + \int_0^t K(t-t')\rho_S(t')dt'$$

where $K(t-t')$ is a memory kernel encoding non-locality in time.

The evolution of an open quantum system can also be described using completely positive trace-preserving (CPTP) maps:

$$\rho_S(t) = \mathrm{E}_t \rho_S(0)$$

where E_t satisfies linearity,

$$\mathrm{E}_t\left(a\rho_1 + b\rho_2\right) = a\mathrm{E}_t\left(\rho_1\right) + b\mathrm{E}_t\left(\rho_2\right)$$

positivity: If $\rho_S(0)$ is positive semi-definite, so is $\mathrm{E}_t\rho_S(0)$ and trace preservation:

$$Tr(\mathrm{E}_t(\rho)) = 1$$

Decoherence describes how quantum coherence is lost due to interaction with the environment. For example, in a two-level system (qubit), decoherence can be modeled by dephasing, in which the off-diagonal elements of the density matrix decay exponentially:

$$\rho_{01}(t) = e^{-\Gamma t}\rho_{01}(0)$$

where Γ is the dephasing rate.

Entropy measures the degree of mixedness in an open quantum system. The von Neumann entropy is defined as:

$$S(\rho) = -Tr(\rho \log \rho)$$

For Markovian dynamics governed by the Lindblad equation, entropy increases over time due to information loss to the environment.

In many open quantum systems, the density matrix evolves toward a steady state, ρ_∞ satisfying

$$\frac{\partial \rho_\infty}{\partial t} = 0$$

For Markovian systems governed by Lindblad dynamics:

$$-\frac{i}{\hbar}\left[H_S, \rho_\infty\right] + \sum_k \left(L_k \rho_\infty L_k^\dagger - \frac{1}{2}\left\{L_k^\dagger L_k, \rho_\infty\right\}\right) = 0$$

Ising model

The Ising model (Glimm & Jaffe, 2012; Hofer-Szabó & Vecsernyés, 2018) is a fundamental mathematical model in statistical mechanics used to study phase transitions and magnetism. It describes a lattice of spins that interact with their neighbors and can exist in one of two states, +1 or -1. The energy (Hamiltonian) of the Ising model is given by:

$$\mathrm{H} = -\sum_{i,j} J_{ij} S_i S_j - H\sum_i S_i$$

where $S_i = \pm 1$ represents the spin at site *i*, J_{ij} is the interaction strength between spins at sites *i* and *j*, *H* is an external magnetic field, and $\langle i, j\rangle$ indicates summation over nearest-neighbor pairs.

The partition function *Z* encodes all thermodynamic properties of the system:

$$\mathrm{Z} = \sum_{\{S_i\}} e^{-\beta \mathrm{H}}$$

where

$$\beta = \frac{1}{k_B T}$$

is the inverse temperature. From the partition function, the free energy *F* is given by

$$F = -k_B T \ln \mathrm{Z}$$

The average magnetization per spin is:

$$\mathrm{M} = \frac{1}{N}\sum_i \langle S_i \rangle$$

where

$$\langle S_i \rangle = \frac{\sum_{\{S_i\}} S_i e^{-\beta \mathrm{H}}}{\mathrm{Z}}$$

The average energy of the system is:

$$E = \langle \mathrm{H} \rangle = -\frac{\partial}{\partial \beta}\left(\ln \mathrm{Z}\right)$$

The specific heat, *C*, measures fluctuations in energy:

$$C = \frac{\partial E}{\partial T} = k_B \beta^2 \left(\langle \mathrm{H}^2 \rangle - \langle \mathrm{H} \rangle^2\right)$$

The spin-spin correlation function, *G*, measures how spins at different sites are correlated:

$$G(r) = S_i S_j, r = |i - j|$$

where

$$G(r) = \begin{cases} 1 & r = 0 \\ e^{\frac{-r}{\xi}} & r > 0 \end{cases}$$

Here, $\xi(T)$ is the correlation length, which diverges near the critical temperature (T_c)

1D Ising model: In one dimension, with nearest-neighbor interactions and no external field (*H*=0), the Hamiltonian simplifies to:

$$H = -J\sum_{i=1}^{N-1} S_i S_{i+1}$$

Transverse field Ising model

The transverse field Ising model (Schmitt *et al.,* 2022; Roberts & Clerk, 2023; Sumeet *et al.,* 2024) is a quantum extension of the classical Ising model. It incorporates quantum mechanics by introducing a transverse magnetic field perpendicular to the spin alignment direction, leading to non-commuting spin operators.

The Hamiltonian for the transverse field Ising model is:

$$H = -J\sum_{i,j} Z_i Z_j - g\sum_j X_j$$

where Z_i and X_i are Pauli matrices acting on site *i* and *j*, representing spin projections along the *z*-axis and the *x*-axis respectively, *J* is the interaction strength between nearest-neighbor spins, *g* is the strength of the transverse magnetic field, and $\sum_{\langle i,j \rangle}$ is the summation over nearest-neighbor pairs.

The spin operators are represented as Pauli matrices:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

These matrices satisfy the anti-commutation relation:

$$\{X, Z\} = XZ + ZX = 0$$

and commute when acting on different lattice sites:

$$[X_i, Z_j] = 0, i \neq j$$

The transverse field introduces non-commutativity between spin projections along *x*- and *z*-axes at the same site:

$$[X_j, Z_j] = 2iY_j$$

Where Y_i is the Pauli matrix for spin projection along the *y*-axis. This non-commutativity makes classical statistical mechanics insufficient to describe the system, requiring a quantum mechanical treatment. For time evolution, the Schrödinger equation is used.

Gate-model quantum computing

Gate-model quantum computing is a framework for performing quantum computations using sequences of quantum gates that act on qubits.

A qubit is the fundamental unit of quantum information, represented as a linear combination of basis states:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Where $|0\rangle, |1\rangle$ are computational basis states, and

$$\alpha, \beta \in C, |\alpha|^2 + |\beta|^2 = 1$$

For a system of *n* qubits, the state resides in a 2^n-dimensional Hilbert space:

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle$$

where c_i are complex coefficients satisfying:

$$\sum_{i=0}^{2^n-1} |c_i|^2 = 1$$

Quantum gates are unitary operators that transform quantum states. A gate acting on a single qubit is represented by a 2 x 2 unitary matrix, while gates acting on multiple qubits are represented by 2^n x 2^n unitary matrices.

Single-qubit gates

(1) A Hadamard gate (*H*) creates superpositions

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Action:

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

(2) Pauli-X gate (quantum NOT) flips the state

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Action:

$$X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$$

(3) Phase gate (*S*) adds a phase to

$$|1\rangle$$

Equation:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

(4) Rotation gates (Rx, Ry, Rz) rotate the qubit around axes:

$$R_x\theta = e^{-i\frac{\theta}{2}X},$$

$$R_y\theta = e^{-i\frac{\theta}{2}Y},$$

$$R_z\theta = e^{-i\frac{\theta}{2}Z},$$

Multi-qubit gates

(1) Controlled NOT (CNOT) gate flips the target qubit if the control qubit is $|1\rangle$

Equation:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(2) A Toffoli gate (CCNOT) is a universal reversible gate acting on 3 qubits.

(3) A swap gate swaps two cubits:

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A quantum circuit is a sequence of quantum gates applied to an initial state. Mathematically, it is represented as:

$$|\Psi_{final}\rangle = U_k U_{k-1} \ldots U_2 U_1 |\Psi_{initial}\rangle$$

where each U_i is a unitary matrix corresponding to a gate. Measurement collapses the state to one of the basis states with probabilities determined by the Born rule:

$$P(|i\rangle) = |\langle i|\Psi\rangle|^2$$

After measurement, the state becomes:

$$|\Psi_{measured}\rangle = |i\rangle$$

where i corresponds to the observed outcome.

Finally, a Quantum Fourier Transform (QFT) transforms a state from the computational basis to the Fourier basis:

$$QFT|j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{\frac{2\pi ijk}{N}} |k\rangle$$

It can be implemented using Hadamard and controlled phase gates.

FORMULATION OF QAML

Quantum Adiabatic Machine Learning (QAML) harnesses quantum adiabatic evolution to train machine learning models, particularly for optimization and classification tasks. The theoretical framework draws upon various concepts from quantum mechanics, statistical mechanics, and classical machine learning, already discussed heretofore.

Classical probability and problem encoding

Classical machine learning often involves minimizing a cost function or energy function $L(\theta)$ where θ represents the model parameters. In QAML, this classical problem is encoded into a quantum Hamiltonian. Consider a binary classification problem with N data points and associated labels,

$$y_i \in \{-1,1\}$$

The classical loss function can be represented like a hinge loss:

$$L(\theta) = \sum_{i=1}^{N} max(0, 1 - y_i f(x_i, \theta))$$

where $f(x_i, \theta)$ is the model's prediction – for example, a linear classifier with $f(x_i, \theta) = \theta \cdot x_i$

The goal is to find

$$\theta^* = argmin_\theta L(\theta)$$

This classical cost function is mapped to a problem Hamiltonian H_p expressed in terms of quantum spin variables. This often involves using the Ising model's energy function to represent the cost landscape. For instance, the model parameters can be mapped to spin variables

$$S_i \in \{-1,1\}$$

associated with qubits

$$H_p = \sum_{i,j} J_{ij} s_i s_j + \sum_i h_i s_i$$

where J_{ij} and h_i are designed such that the ground state of H_p encodes the solution to the classical optimization problem θ^*. The mapping ensures that minimizing H_p corresponds to minimizing $L(\theta)$.

Quantum annealing and adiabatic evolution

Quantum annealing relies on the adiabatic theorem. It can be explained with an initial Hamiltonian H_I that has a known and easily prepared ground state,

$$|\Psi_i\rangle$$

A common choice is a transverse field Ising model:

$$H_I = -g \sum_{i=1}^{N} X_i$$

where X_i is the Pauli-X operator on qubit i, and g is a large transverse field. The ground state is then a uniform superposition:

$$|\Psi_I\rangle = \frac{1}{\sqrt{2^n}} \sum_{x\in\{0,1\}^n} |x\rangle = |+\rangle^{\otimes n}$$

The system is then slowly evolved towards the problem Hamiltonian:

$$H(t) = A(t)H_I + B(t)H_p$$

where $A(t)$ and $B(t)$ are annealing functions with $A(0)=1$ and $B(0)=0$ and $A(T)=0$ and $B(T)=1$, and T is the annealing time. The adiabatic theorem states that if the evolution is slow enough, the system remains in the instantaneous ground state:

$$|\Psi(t)\rangle |GS(t)\rangle$$

The adiabatic condition requires:

$$\frac{\left\langle E_1(t)|\frac{dH(t)}{dt}|E_0(t)\right\rangle}{\left(E_1(t)-E_0(t)\right)^2} << 1$$

where $|E_0(t)\rangle$ and $|E_1(t)\rangle$ are the ground and first excited states of $H(t)$ respectively, and $E_0(t)$ and $E_1(t)$ are their energies. The energy gap

$$\Delta(t) = E_1(t) - E_0(t)$$

is crucial. A smaller gap requires a slower annealing schedule (larger T) to maintain adiabaticity.

Multi-qubit entanglement and quantum resources

Entanglement is crucial for the power of QAML. As the system evolves, the qubits become entangled, allowing the algorithm to explore the complex energy landscape defined by H_p. The state of the system at any time can be expressed as a superposition of multi-qubit states:

$$|\Psi(t)\rangle = \sum_{s_1,s_2,\ldots,s_n\in\{-1,1\}} c(s_1,s_2,\ldots,s_n;t)|s_1,s_2,\ldots,s_n\rangle$$

where $c(s_1,s_2,\ldots,s_n;t)$ are complex amplitudes, and the entanglement entropy can be quantified using the Von Neumann entropy:

$$S = -Tr(\rho_A \log 2\rho_A)$$

where ρ_A is the reduced density matrix for a subsystem A.

Open quantum system effects and mitigation

Real-world quantum annealers are not perfectly isolated and are subject to environmental noise. This leads to decoherence and dissipation, causing the system to deviate from the adiabatic path and populate excited states. We can describe the evolution of the density matrix using a Lindblad master equation:

$$\frac{d\rho}{dt} = -i[H(t),\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

where L_k are Lindblad operators describing environmental interactions (e.g., dephasing, relaxation). The purity of the state, $Tr(\rho)^2$ decreases due to decoherence. Error mitigation techniques, such as dynamical decoupling or quantum error correction, are often employed to counteract these effects. The goal is to maintain a high fidelity in the approximation

$$|\Psi(t)\rangle \approx |GS(t)\rangle$$

Gate-Model emulation

While QAML is typically implemented on dedicated quantum annealers, it can also be emulated on gate-model quantum computers using techniques like Variational Quantum Eigensolver (VQE). VQE approximates the ground state by parameterizing a quantum state with a circuit:

$$|\Psi(\theta)\rangle = U(\theta)|0\rangle^{\otimes n}$$

where $U(\theta)$ is a is a parameterized unitary circuit (variational ansatz) and θ represents the parameters. The energy is then minimized variationally:

$$E(\theta) = \langle\Psi(\theta)|H_p|\Psi(\theta)\rangle$$

The parameters are optimized using classical optimization algorithms.

Relationship to classical annealing and computational complexity

Quantum annealing provides a potential quantum speedup over classical simulated annealing. In classical simulated annealing, the system explores the energy landscape by accepting moves with probability

$$P = exp\left(\frac{-\Delta E}{kT}\right)$$

where $\triangle E$ is the energy difference, k is the Boltzmann constant, and T is the temperature. Quantum annealing, through tunneling and superposition, can potentially overcome energy barriers more efficiently. However, rigorous proofs of quantum speedup for general QAML problems remain a topic of active research. The complexity of the algorithm depends on the minimum energy gap during the adiabatic evolution; a narrow gap can significantly increase the annealing time and negate potential advantages.

QAML is a powerful approach that combines quantum mechanics with machine learning. It involves encoding classical optimization problems into quantum Hamiltonians, leveraging adiabatic evolution and multi-qubit entanglement, and mitigating noise effects. While practical implementations face challenges due to decoherence and hardware limitations, QAML offers a promising avenue for solving complex machine learning problems and underscores the deep interplay between various concepts in physics and computation. The

mathematical formulation provides a rigorous framework for analyzing its capabilities and limitations, paving the way for further advances in quantum-enhanced machine learning.

APPLICATION OF QAML TO PLANT DISEASE DIAGNOSIS

Encoding a plant disease classification problem in quantum Hamiltonians

Considering that the problem involves classifying plants based on disease symptoms encoded in a dataset with feature space X and labels y, such that:

$$X = \{X_1, X_2, \ldots, X_N\}$$

where X_i represents the features (leaf shape, color, texture, etc.), and

$$Y = \{y_1, y_2, \ldots, y_N\}$$

where $y_i \in \{-1,1\}$ indicates disease or healthy plants.

To map this to a quantum system, the problem is encoded in a cost Hamiltonian H_p:

$$H_p = \sum_{i,j} J_{ij} s_i s_j + \sum_i h_i s_i$$

where $s_i \in \{-1,1\}$ are spin variables (representing qubit states), J_{ij} encodes interactions between data points (e.g., similarity in features), and h_i represents local biases (e.g., correlation with class labels).

Minimizing H_p corresponds to finding the optimal configuration of spins $\{s_i\}$ that aligns with the classification goal.

Quantum annealing for model optimization

Using quantum annealing, the quantum system can evolve from an initial Hamiltonian, H_l:

$$H_l = -\sum_i \sigma_i^x$$

where σ_i^x are Pauli-X operators, ensuring the initial ground state is a uniform superposition. The total Hamiltonian evolves as:

$$H(t) = A(t)H_l + B(t)H_p$$

with $A(0)=1$, $B(0)=0$ and $A(T)=0$, $B(T)=1$. The adiabatic theorem ensures that the system remains in its instantaneous ground state, provided the evolution is slow enough:

$$\frac{\left\langle E_1(t) \left| \frac{dH}{dt} \right| E_0(t) \right\rangle}{\left(E_1(t) - E_0(t)\right)^2} << 1$$

where $E_0(t)$ and $E_1(t)$ are the energies of the ground and first excited states, respectively. At $t=T$, the ground state of H_p encodes the optimal spin configuration for classifying diseased and healthy plants.

Quantum feature encoding

Features of the plant dataset can be encoded into quantum states using amplitude encoding:

$$|\Psi\rangle = -\sum_i c_i |x_i\rangle$$

where $c_i = \frac{x_i}{X}$ normalizes the feature vector X. Entanglement can encode correlations between features, enabling the quantum system to model complex dependencies that may indicate plant diseases.

Measurement and classification

After annealing, measuring the final quantum state collapses the system into a configuration $\{s_i\}$. This can be interpreted as:

$$y_i = sign(s_i)$$

where $s_i \in \{-1,1\}$ determines the predicted class (diseased or healthy).

Handling noisy and mixed states

Real-world datasets often contain noise. The system can be modeled as a mixed state, with density matrix:

$$\rho = -\sum_i p_i |\Psi_i\rangle\langle\Psi_i|$$

where p_i represents the probability of state $|\Psi_i\rangle$

The Lindblad master equation governs the open quantum system's evolution:

$$\frac{d\rho}{dt} = -i[H,p] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\left(L_k^\dagger L_k, \rho\right) \right)$$

where L_k are Lindblad operators describing decoherence. This formalism helps model and mitigate errors in quantum systems for robust disease classification.

Quantum advantage

Quantum annealing enables tunneling through energy barriers in the cost landscape, potentially outperforming classical optimization in cases where the energy landscape is rugged and high-dimensional. For plant disease recognition, this can improve classification accuracy and scalability, especially for datasets with complex feature interactions.

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